Ordonancement sous incertitude par la programmation mathématique

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Context Applied scheduling problems face uncertainty due, for instance, to worker performance instabilities and tool quality variations. Robust scheduling has been proposed nearly 20 years ago to handle such uncertainty but it has hardly become a practical tool because the resulting problems are much harder to solve than their deterministic versions. Herein, we study a special case of robust scheduling that characterizes the uncertainty through well-structured sets, U^Γ. Recent results in robust combinatorial optimization have shown that unlike the general case, U^Γ often yields robust problems that are almost as easy as their deterministic counterparts. We will work on extending these positive results to U^Γ-robust scheduling. Hence, the topic studied herein is part of a larger project which aims to make U^Γ-robust scheduling a competitive tool to handle uncertainty.

Problem definition We are given m jobs. Each job has a weight w_j and a processing time p_j. The cost incurred by finishing a job at time t is equal to tw_j, where t is equal to the processing time of job j plus the processing times of all jobs realized before job j. Then, the problem consists in choosing an order for the jobs that minimizes total cost. In this project, we study an uncertain version of the problem where the vector of processing times p can take any value in the following uncertainty set, defined by the mean processing times $\overline{p}_j$, the deviations $\overline{p}_j + \hat{p}_j$, and an integer Γ:

$$U^\Gamma \equiv \left\{ p \in \mathbb{R}^m : p_j = \overline{p}_j + \delta_j \hat{p}_j, j \in \{1, \dots, m\}, \delta_j \in \{0,1\}, j \in \{1, \dots, m\}, \sum_{j=1}^{m} \delta_j \leq \Gamma \right\}.$$

Intuitively, the set considers that at most Γ items will simultaneously take their high processing times. The objective of the robust scheduling problem is to find a schedule on a single machine that minimizes the weighted sum of the completion times for the worst scenario represented by U^Γ.

Objective The robust scheduling problem can be cast as the following min max quadratic program. Let x_{ij} be a binary variable that is equal to 1 iff job i is scheduled prior to job i.

$$\begin{aligned} \min_x \max_{p \in U^\Gamma} \quad & \sum_{i=1}^{m} \sum_{j=i}^{m} p_i w_j x_{ij} \\ \text{s.t.} \quad & x_{ij} + x_{ji} = 1, \quad i, j \in \{1, \dots, m\} \\ & x_{ij} + x_{jk} \leq x_{ik} + 1, \quad i, j, k \in \{1, \dots, m\} \\ & x_{ij} \in \{0, 1\}. \end{aligned}$$

In this internship, we shall see how to efficiently solve the above formulation using well-known linearization techniques from the litterature.

Skills The prospective student should have skills in optimization and programming.

Tools The formulations and algorithms will be coded in the powerful scientific language Julia, developped at the MIT (http://julialang.org/). Julia will be interfaced with the mathematical optimization solver Gurobi (http://www.gurobi.com/).

Some references

- D. Bertsimas and M. Sim. The price of robustness. Operations Research, 52:35–53, 2004.
- Alain Billionnet, Sourour Elloumi, Amélie Lambert: An efficient compact quadratic convex reformulation for general integer quadratic programs. Comp. Opt. and Appl. 54(1): 141-162 (2013)
- Brucker, Peter, and P. Brucker. Scheduling algorithms. Vol. 3. Berlin: Springer, 2007.