Stability-throughput analysis in a multi-hop ad hoc networks with weighted fair queueing

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Abstract—Consider a wireless ad hoc network with random access channel. We present a model that takes into account topology, routing, random access in MAC layer and forwarding probability. We propose a new approach (based on cycle of transmissions) to derive throughput of multi-hop routes and stability of forwarding queues. With this cycle approach, we correct the analytical expressions derived in [2] and discover that their results are valid only in particular cases such as symmetric networks. However, in this paper, we get extended results for general network case. Moreover, we confirm that (i) the forwarding queues in a system of weighted fair queues has a special property and (ii) the end to end throughput of a connection does not depend on the load of the intermediate forwarding queues between a source and a destination. We perform extensive simulations and verify that the analytical results exactly match the results obtained from simulations.

I. INTRODUCTION

A multi-hop wireless ad hoc network is a collection of nodes that communicate with each other without any established infrastructure or centralized control. Each of these nodes is a wireless transceiver that transmits and receives at a single frequency band which is common to all the nodes. These nodes can communicate with each other, however, they are limited by their transmitting and receiving capabilities. Therefore, they cannot directly reach all of the nodes in the network as most of the nodes are outside of direct range. In such a scenario, one of the possibilities for the information transmission between two nodes that are not in position to have a direct communication is to use other nodes in the network. To be precise, the source device transmits its information to one of the devices which is within transmission range of the source device. In order to overcome this, the network operates in a multi-hop fashion. Nodes route traffic for each other. Therefore, in a connected ad hoc network, a packet can travel from any source to its destination either directly, or through some set of intermediate packet forwarding nodes.

Clearly, a judicious choice is required to decide on the set of devices to be used to assist in the communication between any two given pair of devices. This is the standard problem of routing in communication networks. The problem of optimal routing has been extensively studied in the context of wire-line networks where usually a shortest path routing algorithm is used: Each link in the network has a weight associated with it and the objective of the routing algorithm is to find a path that achieves the minimum weight between two given nodes. Clearly, the outcome of such an algorithm depends on the assignment of the weights associated to each link in the network. In the wire-line context, there are many well-studied criteria to select these weights for links, such as delays. In the context of wireless ad-hoc networks, however, not sufficient attempts have been made to (i) identify the characteristics of the quantities that one would like to associate to a link as its weight, and in particular (ii) to understand the resulting network performance and resource utilization. Some simple heuristics have been frequently reported to improve performance of applications in mobile ad-hoc networks (see [9] and reference therein).

To study this problem, we consider in this paper the framework of random access mechanism for the wireless channel where the nodes having packets to transmit in their transmit buffers attempt transmissions by delaying the transmission by a random amount of time. This mechanism acts as a way to avoid collisions of transmissions of nearby nodes in the case where nodes can not sense the channel while transmitting (hence, are not aware of other ongoing transmissions). We assume that time is slotted into fixed length time frames. In any slot, a node having a packet to be transmitted to one of its neighboring devices decides with some fixed (possibly node dependent) probability in favor of a transmission attempt. If there is no other transmission by the other devices whose transmission can interfere with the node under consideration, the transmission is successful. We assume throughout that there is some mechanism that notifies the sender of success or failure of its transmissions. For example, the sources get the feedback on whether there was zero, one or more transmissions (collision) during the time slot.

At any instant in time, a device may have two kinds of packets to be transmitted:

1) Packets generated by the device itself. This can be sensed data if we are considering a sensor network.
2) Packets from other neighboring devices that need to be forwarded.

In this paper we consider two separate queues for these two types and do a weighted fair queueing (WFQ) for these two queues. This type of configuration allow us to include in the model the cooperation level which represents the fraction of the traffic forwarded by a node in ad-hoc network.

Several studies have focused on wireless network stability and finding the maximum achievable throughput. Among the most studied stability problems are scheduling [11], [12] as well as for the Aloha protocol [1], [10], [14]. Tassiulas and Ephremides [11] obtain a scheduling policy for the nodes that maximises the stability region. Their approach inherently avoids collisions which allows to maximize the throughput. Radunovic and Le Boudec [5] suggest that considering the total throughput as a performance objective may not be a good objective. Moreover, most of the related studies do not consider the problem of forwarding and each flow is treated similarly (except for Radunovic and Le Boudec [5], Huang and Bensaou [8] or Tassiulas and Sarkar [13]). Our
setting is different than the mentioned ones in the following: the number of retransmission is finite, and therefore in our setting, the output and the input rates need not be the same. In recent past year, there has been a considerable effort on trying to increase the performance of wireless ad hoc networks since Gupta and Kumar [7] showed that the capacity of a fixed wireless network decreases as the number of nodes increases. Grossglauser and Tse [6] presented a two-phase packet forwarding technique for mobile ad-hoc networks, utilizing the multiuser diversity, in which a source node transmits a packet to the destination when this destination becomes the closest neighbors of the relay. This scheme was shown to increase the capacity of the MANET, such that it remains constant as the number of users in the MANET increases.

In [2], working with the above mentioned system model, we have already studied the impact of routing, channel access rates and weights of the weighted fair queueing on throughput, stability and fairness properties of the network. We obtained important insights into various tradeoffs that can be achieved by varying certain network parameters. Some of the important results are that

1) As long as the intermediate queues in the network are stable, the end-to-end throughput of a connection does not depend on the load on the intermediate nodes.
2) Routing can be crucial in determining the stability properties of the network nodes. We showed that if the weight of a link originating from a node is set to the number of neighbors of this node, then shortest path routing maximizes the minimum probability of end-to-end packet delivery.
3) The results of this paper extended in a straightforward manner to systems of weighted fair queues with coupled servers.

Our contribution in this paper consists on the following:

1) We identify that the departure and the arrival rate expressions derived in paper [2] are not correct. However, their conclusions remain true and their analytical results are valid only for some particular cases such as symmetric networks. We show these cases in the discussion of section III.
2) We use a new approach to correct and derive the expressions of the arrival and departure rates, so we get extended results that works on general cases. Our new approach consists on introducing the notion of cycle of transmissions where a cycle is the number of slots needed to transmit successfully or to drop a packet from the MAC layer. With this approach, the study of the problem is simplified and a discussion about the stability-throughput tradeoff is performed.
3) We validate our findings with simulations.

In section II, we present the cross-layer network model with the new approach of cycle of transmissions. In section III, we write the balance rate equations from which appears the property of the forwarding queues and the throughput independency from the forwarding weight of the WFQ. The validation of analytical results is done with a discrete time simulator in section IV. Finally, we end with a conclusion in section V.

II. Network Model

A. Assumptions and definitions

We model the ad hoc wireless network as a set of N nodes deployed arbitrarily in a given area. We number the nodes with integer numbers.

We assume the following:

- **A one simple channel**: Nodes use the same frequency for transmitting with an omni-directional antennas. A node j receives successfully a packet from a node i if and only if there is no interference at the node j due to another transmission on the same channel. A node cannot receive and transmit at the same time.
- **Discrete time scale**: the time is divided into a fixed length slots: \( t \) and \( t + \delta \) correspond to the beginning of two consecutive slots where \( t \) is an integer indicated the time and \( \delta \) is the length of a slot. We assume that one unit slot is needed to transmit a packet from one node to another. All transmissions in a time slot begins at the beginning of the slot and are received at the end of the same slot.
- **Two types of queues**: two queues are associated with each node. The first one is the forwarded queue, noted by \( F_i \) (proper to the node i), which carry all the packets originated from a given source and destined to a given destination. The second is \( Q_i \) which carries the proper packets of the node i (in this case \( i \equiv s \) where s designates a source node). We assume that each node has an infinite capacity of storage for the two queues. Packets are served with a first in first served fashion. When \( F_i \) has a packet to be sent, the node chooses to send it from \( F_i \) with a probability \( f_i \). In other terms, it chooses to send from \( Q_i \) with probability \( 1 - f_i \). When one of these queues is empty then we choose to send a packet from the none empty queue with a probability 1.
- **Saturated network**: each node has always packets to be sent from queue \( Q_i \), whereas \( F_i \) can be empty. Consequently, the network is considered saturated and depends on the channel access mechanism.

B. Network Layer

Network layer handles the two queues \( Q_i \) and \( F_i \) using the WFQ scheme, as described previously. Also, this layer maintains routing algorithms. So, each node acts as a router, it permits to relay packets originated from a source s to a destination d. It must carries a routing information which permits sending of packets to a destination via a neighbor. In this paper, we assume that nodes form a static network where routes between any source s and destination d are invariant in the saturated network case. Proactive routing protocols as OLSR (Optimized Link State Routing) construct and maintain a routing table that carry routes to all nodes on the network. These kind of protocols correspond well with our model. Let \( R_{s,d} \) be the set of nodes between a node s and d.

C. MAC Layer

We assume a channel access mechanism only based on a probability to access the network i.e. when a node i has a packet to transmit from the queue \( Q_i \) or \( F_i \), it accesses the channel with a probability \( P_i \). It can be similar to CSMA/CA or any other mechanism to access the channel. For example, with the IEEE 802.11, \( P_i \) depends on
the number of neighbors, on the back off mechanism and the probability of collision, see [3], [4]. Also the exposed terminal impact with the IEEE 802.11 or more generally the impact of the channel access mechanism is included implicitly in the value of $P_t$. The scheduler of transmission overall the network depends on $P_t$. We assume that each node is notified about the success or failure of its transmitted packets. A packet is failure only when there is a collision on the intended receiver. We have considered previously infinite buffer size, therefore, there is no packet loss due to overflow at the queues. The only source of packet loss is due to collisions. For a reliable communication, we allow a limit number of successive transmissions of a single loosed packet, after that it will be dropped definitively. We denote $K_{i,s,d}$ the maximum number of successive collisions allowed of a single packet sent from the node $i$ on the path from $s$ to $d$. $K_{i,s,d}$ is known as the limit number of retry. Let also $L_{i,s,d}$ be the expected number of attempts till successful or a definitely drop from node $i$ on the path from $s$ to $d$.

D. Cross-Layer Representation of the Model

The model of figure 1 represents our model in this paper. The two layers are clearly separated. Attempting the channel begins by choosing the queue from which a packet must be selected. And then, this packet is moved from the corresponding queue from the network layer to the MAC layer where it will be transmitted and retransmitted, if needed, until its success or drop. In this manner, when a packet is in the MAC layer, it is itself attempted successively until it is removed from the node.

We present the model for each node $i$, as an axe of time graduated by time slots and cycles. A cycle is defined as the number of slots needed to transmit a single packet until its success or drop. We distinguish two types of cycles: The forwarding cycles in relief to the packets of $F_i$ and the source cycles in relief to the packets coming from $Q_i$. Also, each cycle is affected to a connection. The beginning of each cycle represents the choice of the queue from which we choose a packet and the choice of the connection where to send it. Whereas, the slots that the cycle represents the attempts of the packet itself to the channel, including its retransmissions. Hence, the distinction of the network and MAC layer is now clear.

We need to define formally the model, so we will be able to derive some formulas in the next sections. For that, consider the following counters:

- $C_{t,i}$ is the number of cycle of the node $i$ till the $t^{th}$ slot.
- $C_{t,i}^f$ (resp. $C_{t,i}^s$) is the number of all forwarding cycles (resp. source cycles) of the node $i$ till the $t^{th}$ slot.
- $C_{t,i,s,d}^f$ (resp. $C_{t,i,s,d}^s$) is the number of forwarding cycles (resp. source cycles) corresponding to the path $R_{s,d}$ of the node $i$ till the $t^{th}$ slot.

- $T_{t,i,s,d}$ is the number of times we found at the first slot of a cycle and at the first position in the queue $F_i$ a packet for the path $R_{s,d}$ of the node $i$ till the $t^{th}$ slot.
- $I_{t,i,s,d}$ is the number of cycles corresponding to the path $R_{s,d}$ of the node $i$, and where a cycle is ended by a success of the transmitted packet, till the $t^{th}$ slot.
- $A_{t,i,s,d}$ is the number of arrival packets to node $i$ on the path $R_{s,d}$.

Figure 2 shows a simple example with some numerical values of the previous counters for a single node $i$.

E. Main Notations

We summarize principle notations of the paper in the following two lists:

1) MAC layer notations:

- $P_t$ is the probability of transmission on the channel of the node $i$.
- $P_{i,s,d}$ is the probability that a transmission from node $i$ on the path from $s$ to $d$ is successful.
- $K_{i,s,d}$ is the maximum number of successive collision (or failure) allowed of a single packet sent from the node $i$ on the path from $s$ to $d$. After a $K_{i,s,d}$ failure, the packet is dropped i.e. it is removed from the node $i$.
- $L_{i,s,d}$ is the expected number of attempts till successful or a drop from node $i$ on the path from $s$ to $d$.

2) Network layer notations:

- $f_i$ is the probability to send a packet from the queue $F_i$ when it carries a packet.
- $N(i)$ is the set of neighbors of the node $i$. We assume that all the nodes in $N_i$ has $i$ as a neighbor.
- $R_{s,d}$ is the set of intermediate nodes in a path between a node $s$ and a node $d$. $s$ and $d$ are not in this set.
- $R_{i,s,d}$ is the set of nodes $R_{s,i} \cup i$ in the path $s, d$.
- $j_{i,s,d}$ designates a neighbor node of $i$ that comes after $i$ in the set $R_{s,d}$ toward the destination on the path from $s$ to $d$. It is the next hop node of the node $i$.
- $\pi_i$ is the probability that the queue $F_i$ has at least one packet to be forwarded in the beginning of each cycle.
- $\pi_{i,s,d}$ is the probability that the queue $F_i$ has a packet at the first position ready to be forwarded to the path $R_{s,d}$ in the beginning of each cycle. Then, $\pi_i = \sum_{s,d,i R_{s,d}} \pi_{i,s,d}$.
- $P_i$ is the probability that the node $i$ chooses the path $R_{i,d}$ (whose destination is $d$) for sending
packets from $Q_i$. Normally, this parameter can be assigned to the node $i$ application layer decision. $P_{i,s,d}$ and $L_{i,s,d}$ are derived in section III. $R_{s,d}$, $R_{i,s,d}$, $f_i$ and $N(i)$ are given from the routing protocol. Whereas $\pi_i$, the indication of the load on $F_i$, is to be found from the rate balance equations on section III.

III. Stability Properties of the Forwarding Queues

Our main objective in this section is to derive the rate balance equations from which some properties of the forwarding queues can be deduced. For that we need to write the departure rate from each node $i$ and the end to end throughput between a couple of nodes.

From a practical point of view, each node owns three main parameters $P_i$, $K_{i,s,d}$ and $f_i$, that can be managed and set in such a way that each node can maintain stability, or the end to end throughput on a path can be optimized. In this paper, by fixing the routing paths (from $R_{s,d}$ and route choice (from $P_{s,d}$), we will observe forwarding queue stability function of these later three main parameters.

Giving a saturated network case where each node has a packet on its queue $Q_i$ and attempts transmitting all the time to the channel, the forwarding queue $F_i$ of each node will have a $\pi_i$ load when it tries to forward packets to its neighbors. $\pi_i$ appears to have a major information on stability and needs to be determined function of node main parameters.

A. The Rate Balance Equations

The forwarding queue $F_i$ is stable if the departure rate of packets from $F_i$ is equal to the arrival rate into it. This is a simple definition of stability that can be written with a rate balance equation. In this paper, we are going to derive this equation for each node $i$ using the cycle approach of the model in figure 1. In fact, it is judicious to write the rate balance equation of node $i$ for each connection and then do the summation for all others. For that, we proceed by determining the corresponding formulas as following:

1) $P_{i,s,d}$ and $L_{i,s,d}$: A transmission from $i$ to $j$, $s,d$ is successful if neither $j$, $s,d$ nor any of its neighbors, except $i$, is transmitting on the same slot. Therefore, $P_{i,s,d}$ can be written in terms of the probability of transmission.

$$P_{i,s,d} = (1 - P_{j,i,s,d}) \prod_{j \in N(i,s,d) \setminus i} (1 - P_j)$$ (1)

$L_{i,s,d}$ can be formulated in terms of $K_{i,s,d}$ and $P_{i,s,d}$ as following:

$$L_{i,s,d} = \frac{1 - (1 - P_{i,s,d})^{K_{i,s,d}}}{P_{i,s,d}}$$

2) The Departure Rate: The probability that a packet is removed from a node $i$ by a successful transmission or a drop (i.e. a successive $K_{i,s,d}$ Failure) is the departure rate from $F_i$. We denote it by $d_i$. The departure rate concerning only the packets sent on the path $R_{s,d}$ is denoted $d_{i,s,d}$. Formally, for any node $i$, $s$ and $d$ such that $R_{s,d} > 0$ and $i \in R_{s,d}$, the long term departure rate of packets from node $i$ on the route from $s$ to $d$ is

$$d_{i,s,d} = \lim_{t \to \infty} \frac{C_{t,i,s,d}}{t} = \lim_{t \to \infty} \frac{T_{t,i,s,d} C_{t,i,s,d}}{C_{t,i}} = \lim_{t \to \infty} \frac{T_{t,i,s,d} C_{t,i}}{C_{t,i,s,d} t}$$ (2)

- $\frac{T_{t,i,s,d}}{C_{t,i,s,d}}$ is exactly the probability that $F_i$ carried a packet to the path $R_{s,d}$ in the beginning of each cycle. Therefore, $\frac{T_{t,i,s,d}}{C_{t,i,s,d}} = \pi_{i,s,d}$. 
- $\frac{C_{t,i,s,d}}{C_{t,i}}$ is exactly the probability that we have chosen a packet from $F_i$ to be sent when $F_i$ carried a packet to the path $R_{s,d}$ in the first position and in the beginning of a forwarding cycle. Therefore, $\frac{C_{t,i,s,d}}{C_{t,i}} = f_i$. 
- $\frac{t}{C_{t,i}}$ is the average length in slots of a cycle of the node $i$. A cycle length on the path $R_{s,d}$ is formed by the attempt slots that does not lead to a channel access and the transmission and retransmissions of the same packet until a success or a drop. Thus a cycle length for a one path $R_{s,d}$ of a node $i$ is $\frac{C_{t,i}}{F_i}$. When a node transmits to several paths, we need to know the average cycle length. This given by $\frac{T_{t,i}}{L_i}$ where $L_i$ is the average of $L_{i,s,d}$ of these paths. $L_i$ is given by:

$$L_i = \sum_{s,d,i \in R_{s,d}} \pi_{i,s,d} f_i L_{i,s,d} + \sum_{d} (1 - \pi_i f_i) P_{i,d} L_{i,i,d}$$

Therefore, $\frac{C_{t,i}}{L_i} = \frac{P_{i,s}}{L_i}$. Consequently,

$$d_{i,s,d} = \pi_{i,s,d} f_i P_{i,s} \frac{L_i}{L_i}$$ (3)

It is clear that the the departure rate $d_{i,s,d}$ on a path $R_{s,d}$ of a node $i$ does not depend on the parameters of only one path but it is also related to the expected number of transmissions to all other paths used by node $i$. This dependency appears in $L_i$ which is not introduced in paper [2]. Moreover, it is easy to derive the total departure rate $d_i$ on all paths:

$$d_i = \sum_{s,d,i \in R_{s,d}} d_{i,s,d} = \pi_i f_i P_i \frac{L_i}{L_i}$$ (4)

3) The Arrival Rate: The probability that a packet arrives to the queue $F_i$ of the node $i$ is the arrival rate that we denoted it by $a_i$. When this rate concerns only packets sent on the path $R_{s,d}$, we denoted it by $a_{i,s,d}$. Formally, for any node $i$, $s$ and $d$ such that $R_{s,d} > 0$ and $i \in R_{s,d}$, the long term arrival rate of packets into $F_i$ for $R_{s,d}$ is

$$a_{i,s,d} = \lim_{t \to \infty} \frac{A_{t,i,s,d}}{t}$$ (5)

$$= \lim_{t \to \infty} \frac{C_{t,s} T_{t,i,s,d} C_{t,s} A_{t,i,s,d}}{C_{t,i} C_{t,s} T_{t,i,s,d} I_{t,i,s,d}}$$ (6)

- $\frac{C_{t,s}}{C_{t,i}} = 1 - \frac{C_{t,s}}{C_{t,i}} = 1 - \pi_s f_s$, this is exactly the probability to get a source cycle i.e. to send a packet from the queue $Q_s$. 
- $\frac{C_{t,s}}{C_{t,i}}$ is the probability to choose the path $R_{s,d}$ to send a packet from $Q_s$. Therefore, $\frac{C_{t,s}}{C_{t,i}} = P_{s,d}$. 
- $\frac{C_{t,i}}{C_{t,i,s,d}} = \frac{R_{s,d} I_{i,s,d}}{L_i}$ is the probability that a source cycle on the path $R_{s,d}$ ends with a success i.e. the packet sent from $Q_s$.
is received on the queue $F_{s,s,d}$. Therefore, $\frac{L_{s,s,d}}{C_{s,s,d}} = (1 - (1 - P_{s,s,d})K_{s,s,d})$.

- $A_{i,s,d}$ is the probability that a packet received on the node $j_{s,s,d}$ is also received on the queue $F_i$ of the node $i$. For that, this packet needs to be received by all the nodes in the set $R_{i,s,d}$. Therefore, $A_{i,s,d} = \prod_{k \in R_{i,s,d}} (1 - (1 - P_{k,s,d})K_{k,s,d})$.

Consequently,

$$a_{i,s,d} = (1 - \pi_s f_i) P_{s,d} \frac{P_s}{P_i} \prod_{k \in R_{i,s,d} \setminus i} (1 - (1 - P_{k,s,d})K_{k,s,d})$$

(7)

Remark that when the node $i$ is the destination of a path $R_{s,d}$, then $a_{d,s,d}$ represents the end to end average throughput of a connection from $s$ to $d$. Also, note that the global arrival rate is: $a_i = \sum_{s,d \in R_{s,d}} a_{i,s,d}$

4) The rate balance: Finally, in the steady state if all the queues in the network are stable, then for each $i$, $s$ and $d$ such that $i \in R_{s,d}$ we get $a_{i,s,d} = a_{s,d}$, which is the rate balance equation on the path $R_{s,d}$.

Let $y_i = 1 - \pi_i f_i$ and $z_{i,s,d} = \pi_{s,d} f_i$. Thus $y_i = 1 - \sum_{s,d \in R_{s,d}} z_{i,s,d}$. Then rate balance equation becomes,

$$\sum_{d \in R_{s,d}} z_{i,s,d} = y_i (\sum_{s',d'} z_{i,s',d'} L_{i,s',d'} + \sum_{d'} y_{i,s'} L_{i,s',d'} w_{s,i})$$

(8)

where

$$w_{s,i} = \sum_{d \in R_{s,d}} \frac{P_{s,d} P_s}{P_i} \prod_{k \in R_{i,s,d} \setminus i} (1 - (1 - P_{k,s,d})K_{k,s,d})$$

B. Discussion

We are interested to find the unknowns ($z_{i,s,d}$ and $y_i$) of the system of equations 8 for all $i$, $s$ and $d$, function of the network parameters. In fact, $y_i$ represents the probability to send packets from the queue $Q_i$ of the node $i$ to send the own packets of node $i$. More formally, it represents the rate of the source cycles. When $f_i$ is fixed, it is somehow a measure of the load on the forwarding queue $F_i$. When $y_i$ has a small value, it informs about a high load on $F_i$ and about a large opportunity to send packets from $F_i$ so to maintain stability. In this situation i.e for small $y_i$, the own packets of node $i$ have less priority than $F_i$ packets and their transmissions are delayed.

1) Particular case: The system of equations (8) for all $i$, $s$ and $d$ is not always linear and the solution is not so easy to find. However, there are some cases where this system is linear. It can be obtained when for each node $i$ ($0 \leq i \leq N$), we have that $T_i$ is independent from the unknowns $y_i$ and $z_{i,s,d}$. In other terms, we need $T_i = L_{i,s,d}$ for all $s,d$ : $i \in R_{s,d}$ and for all $0 \leq i \leq N$. A symmetric network with $N(i) = n$, $P_i = P$ and $K_{i,s,d} = K$ is an example of this case. Consequently, the system from equation 8 can be written as

$$1 - y_i = \sum_s y_s w_{s,i}$$

(9)

where

$$w_{s,i} = \sum_{d \in R_{s,d}} \frac{P_{s,d} P_s P_{s,d} L_{i,s,d}}{P_i} \prod_{k \in R_{i,s,d} \setminus i} (1 - (1 - P_{k,s,d})K_{k,s,d})$$

Therefore, the system of equations (9) can be written in a matrix form as following and resolved easily:

$$y(I + W) = 1$$

where $W$ is an $N \times N$ matrix whose $(s,i)^{th}$ entry is $w_{s,i}$ (independent on $y_i$) and $y$ is an $N$-dimensional row vector.

Furthermore, when considering the general system of equations (8), and when a node uses the same neighbor as a next hop to forward all its packets, the $y_i$ of this node can match the one found by the system of paper [2]. Numerical results of section IV will verify this conclusion on a given asymmetric network. In addition, a high node density of a network and uniform traffic distribution can lead to a system that can be approximated and solved using a linear system. But in general case, we need to resolve a non linear one.

2) Implications: In the following, we present some implications of the global rate balance.

- When a valid solution of the system of equations 8 for all $i$, $s$ and $d$ exists, then each $y_i$ and $z_{i,s,d}$ does not depend on any forwarding probability. In fact, the equation 8 is independent on $f_i$, i.e there is no $f_i$ out of the unknowns ($y_i$ and $z_{i,s,d}$), thus the solution will also be independent. This independency is an interesting property of the forwarding queues. Hence, $y_i$ depends only on the routing, $P_{s,d}$, the probability of transmission and the maximum number of attempts.

- The condition of stability of a forwarding queue is simply $0 \leq y_i < 1$ i.e $0 \leq 1 - y_i \leq 1$. It implies that when we choose $f_i \in (1 - y_i, 1)$ for all $i$, the network will maintain its stability only for the values of $y_i$ that respect the condition $0 < y_i \leq 1$. It is an interesting network design to find a set of forwarding probability that guarantees stability on the network.

- The expression of the throughput between a couple of nodes, deduced from the equation 7, is mainly formed by $y_s$ ($s$ indicates the source node of a connection) and the end to end probability of success. It does not depend on the forwarding probabilities of the intermediate nodes between the source and the destination. Hence, the load on the intermediate forwarding queues on a given path does not affect the throughput. However, $y_s$ has an important role on the intensity of the throughput.

- There is no tradeoff between throughput and stability caused by the forwarding probability. A real tradeoff is caused by the maximum number of attempts: the throughput is ameliorated when reattempting many times on a path, while the service rate on a forwarding queue is slowed down causing low stability region.

- There is no throughput-delay tradeoff when varying the forwarding probability. In other terms, when the nodes increase their forwarding probability the end to end delay of a connection will be ameliorated without affecting the throughput.
IV. NUMERICAL RESULTS AND SIMULATIONS

In this section, we present some numerical results and validate the expressions found in the previous sections with a discrete time network simulator. We have implemented this simulator according to the model of section II. Hence, it appears to be a valuable tool of measurement.

We deploy an asymmetric static wireless network with 11 nodes as shown in figure 3.

![Wireless Network Diagram](image)

Fig. 3. Wireless network

Five connections are established a, b, c, d and e as indicated in the same figure (a dashed or complete line between two nodes in this figure means that there is a neighboring relation). These connections choose the shortest path in terms of hops to route their packets. We choose the parameters $K_{i,s,d} = K$, $f_i = f$ and $P_i$ in a manner of enabling stability, for all $i, s$ and $d$. We fix $f = 0.8$ except contraindication. Let $P_2 = 0.3, P_3 = 0.3, P_4 = 0.4, P_5 = 0.5, P_7 = 0.3, P_8 = 0.3, P_{10} = 0.4$ be the fixed transmission probabilities for nodes 2, 3, 4, 5, 7, 8 and 10 while $P_i = P$ for all other $i$. Many nodes need to have a fix transmission probabilities so to get a stable queues for all nodes.

![Forwarding Probability Limit](image)

Fig. 4. Forwarding probability limit

We start by looking at the numerical results of the forwarding probability limit allowed to maintain stability. This is shown in figure 4. For $K = 4$, the stability region is already small. When it is allowed to retransmit a lot of times, the waiting in the queue get larger. For that we need more forwarding aptitude to avoid instability. From figure 4, each node must choose a forwarding probability higher than the curves limit for the two cases $K = 1$ and $K = 4$.

The $y_i$ solution is shown in figure 5 in terms of the transmission probability $P$. It is the aptitude that a node $i$ sends its own packets from the queue $Q_i$. We see that the node 10 for a $P$ around 0.55 is obliged to forward a lot of packets to its neighbors to avoid instability. From figures 6 to 9, we show the throughput of the five connections and the $\pi_i$ for all $i$ where $\pi_i = 1 - y_i f$. Figures 6 and 8 are numerical results and figures 7 and 9 are from simulations. It is clear from these figures that the mathematical expressions match exactly the simulator result.

![Simulation Throughput](image)

Fig. 7. Simulation throughput for $K = 4$ and $f = 0.8$
rates from and to a node $i$ on a given path $R_{s,d}$ does not depend on all the neighbors of $i$. In the asymmetric network of figure 3, all $\pi_i$ matches as in our model except for the nodes 4, 5, and 8 where several connections and different next hop nodes are used. Figure 11 shows the comparison between the solution given from the linear and non linear system for nodes 4, 5 and 8. Usually, $\pi_i$ of each node depends on the load of other nodes in the network. In our network case, nodes (2, 3, 7 and 10) that use the same next hop node (i.e. $L_{ij} = L_{i,s,d}$) to route packets have not been influenced with the changes of $\pi_4$, $\pi_5$ and $\pi_8$. For example, node 3 has node 8 as a source on the connection $d$, but no influence has been observed on $\pi_3$ when comparing the solution of the linear and non linear system. This remark remains an opened question.
From figure 12 to 14, we vary the load of the forwarding queues by changing the forwarding probability of each node. The parameter of the network are the same as before except that we fix $P = 0.2$ and vary $f$ for all nodes except for node 5 and 10, we set $f_5 = 0.8$ and $f_{10} = 0.7$. We observe that when $f$ is small the system is not stable, more precisely the nodes 2, 3, 4, 7 and 8 are suffering from a congestion as shown in figure 12. They need to deliver more packets from the forwarding queue in a faster manner. In this unstable case, the throughput of all connections is sensitive with the $f$ variation, and it increases with $f$ (see figure 13) until the system becomes stable around $f = 0.4$. The $y_i$ as shown in figure 14 remains also independent of $f$ in the stability region. Consequently, the throughput that does depend on $y_i$, is also independent of $f$, thus it is independent of the load $\pi_i$ of the nodes engaged in a connection.

All the above analysis and simulations were done in the saturated case where each node has always a packet to transmit from its queue $Q$. We were wondering if the forwarding probability has an impact in the unsaturated case. For that, we consider that packets generated in each node arrive to the queue $Q$ following a geometric process with mean $\lambda$. We show in figure 15 the connection throughput as function of the forwarding probability for different $\lambda$. It appears that the throughput reaction is similar to the saturated case. It is independent of the weight given to the forwarding queue when the stability is reached for a given $f$. Moreover, it exists an optimal $\lambda$ in the interval $[0.01, 0.1]$ as shown in the same figure that maximizes the throughput. For a very small $\lambda$ the success probability of packets is very high and for high $\lambda$ there are many collisions.

V. CONCLUSION

In this paper, we have presented a cross-layer model that takes into account many parameters concerning network and MAC layer. By separating clearly the role of each layer with the cycle of transmissions approach, we correct the analytical expressions of [2] and discover that their results can be valid only in particular cases. Moreover, we confirm that forwarding queues does not depend on the forwarding probability which is the weight of the WFQ. As a consequence, the end to end throughput between a couple of nodes does not depend on the load of the intermediate forwarding queues. We have performed numerical results and simulations to validate our findings. A perspective of this work is to consider the unsaturated case and study analytically the impact of the forwarding probability on the performances of the network.

REFERENCES